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A TYPE OF LINEAR GAME WITH MIXED CONSTRAINTS ON THE CONTROLS*

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A game is considered in which the first person's control is subject to a geometric and an integral constraint. The second person can choose a control, subject to the geometric constraint. The game ends at a given instant. The sets of controls values and the terminal set are of the same type. The conditions are found for the game to end from a given initial position. The player's controls are constructed, and an example is given.

Differential games with mixed constraints were considered in /1, 2/. The present types of game may not satisfy the regularity condition /2, 3/.

1. A linear differential game with a fixed instant of termination can be reduced by a linear change of variables /3/ to a game with simple motion.

We consider the game whose equations of motion are

$$z^{\cdot} = -\alpha(t)u + \beta(t)v, \quad z \in R^n \tag{1.1}$$
$$\nu^{\cdot} = -|u|, \quad |u| \leqslant \gamma, \quad |v| \leqslant 1, \quad \gamma > 0$$

Here, $|x|$ is the norm of the vector $x \in R^n$, α and β are continuous non-negative functions for $t \leqslant p$, where p is the instant of termination. The second equation in (1.1) characterizes the law of variation of the resources expended on forming the control u. The choice of this control is subject to the condition $\nu \geqslant 0$.

In the phase space of the game $Z = \{(z, \nu): z \in R^n, \ \nu \geqslant 0\}$ we are given the terminal set

$$X = \{(z, \nu): \ |z| \leqslant \varphi(\nu), \ \nu \geqslant \delta\}, \quad \delta \geqslant 0 \tag{1.2}$$

For instance, if the termination conditions are specified by $|z(p)| \leqslant a$, then $\varphi(\nu) = a$ and $\delta = 0$.

We assume that the continuous function φ is non-negative and does not decrease for $\nu \geqslant \delta$.

We will write the operator of program absorption /4/ for terminal set (1.2). The position $(z, \nu) \in Z$ belongs to the set $T_\tau^p(X)$ if and only if, given any control $|v(t)| \leqslant 1$, measurable in the interval $[\tau, p]$, there is a control $|u(t)| \leqslant \gamma$, measurable for $\tau \leqslant t \leqslant p$, such that

$$\left| z + \int_\tau^p (-\alpha(t)u(t) + \beta(t)v(t))\,dt \right| \leqslant \varphi(\nu(p)), \tag{1.3}$$
$$\nu(p) = \nu - \int_\tau^p |u(r)|\,dr \geqslant \delta$$

With $\nu \geqslant 0$ we put

$$f(p, \tau, \nu) = \max \int_\tau^p \alpha(t)w(t)\,dt, \quad \int_\tau^p w(t)\,dt \leqslant \nu, \quad 0 \leqslant w(t) \leqslant \gamma \tag{1.4}$$

We can then obtain from (1.3):

$$T_\tau^p(X) = \{(z, \ \nu): |z| \leqslant \psi(\tau, \ \nu), \ \nu \geqslant \delta\} \tag{1.5}$$

$$\psi(\tau, \nu) = \max(\varphi(\nu - \mu) + f(p, \tau, \mu)) - \int_\tau^p \beta(t)\,dt, \quad 0 \leqslant \mu \leqslant \nu - \delta \tag{1.6}$$

The maximum with respect to μ is reached in (1.6), since function (1.4) is continuous with respect to ν. For, regarding (1.4) as a problem of moments, we obtain /5/

$$f(p, \tau, \nu) = \max_I \int_I \gamma\alpha(t)\,dt, \quad \sigma(I) = \min(p - \tau, \nu/\gamma) \tag{1.7}$$

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Here, $\sigma(I)$ is the measure of the set $I \subset [\tau, p]$. It follows from (1.7) that, for $0 \leqslant \nu_2 < \nu_1$,

$$0 \leqslant f(p, \tau, \nu_1) - f(p, \tau, \nu_2) \leqslant (\nu_1 - \nu_2) \max_t \alpha(t), \quad \tau \leqslant t \leqslant p \tag{1.8}$$

We consider an initial point (z, ν), $\nu \geqslant \delta$, which does not belong to set (1.5). We construct a second player's control which guarantees that set (1.2) is not hit at instant p.

We put

$$v(t) = z/|z|, \quad z \neq 0; \quad |v(t)| = 1, \quad z = 0 \tag{1.9}$$

It then follows from the equations of motion (1.1) that, with any control $|u(t)| \leqslant \gamma$ and any $t_1 > \tau$,

$$|z(t_1)| \geqslant |z| + \int_\tau^{t_1} \beta(t)\, dt - \int_\tau^{t_1} \alpha(t) |u(t)|\, dt \tag{1.10}$$

Hence, using the inequality $|z| > \psi(\tau, \nu)$ and the definition of function (1.4), we obtain $|z(p)| > \varphi(\nu(p))$.

2. Consider the possibility of moving the position at instant p into set (1.2), from an initial state belonging to set (1.5).

The first person's strategy /3/ will be sought in the form

$$\begin{aligned} &u(t, z) = w(t)\, U(z), \quad 0 \leqslant w(t) \leqslant \gamma \\ &U(0) = \{z\colon |z| \leqslant 1\}; \quad U(z) = z/|z|, \quad z \neq 0 \end{aligned} \tag{2.1}$$

For the initial condition $z(\tau) = z$, $\nu(\tau) = \nu$, and any functions $0 \leqslant w(t) \leqslant \gamma$ and $|v(t)| \leqslant 1$, measurable in the interval $[\tau, p]$, we understand by the motion $z(t)$, $\nu(t)$ any solution of the differential inclusion

$$\begin{aligned} &z^{\cdot}(t) \in -\alpha(t)\, w(t)\, U(z(t)) + \beta(t)\, v(t); \quad z(\tau) = z \\ &\nu^{\cdot}(t) = -w(t), \quad z(t) \neq 0; \quad \nu^{\cdot}(t) \in [-w(t), 0], \ z(t) = 0, \\ &\nu(\tau) = \nu \end{aligned} \tag{2.2}$$

For any $z, \tau \leqslant t \leqslant p$, the right-hand side of inclusion (2.2) is a convex compactum, which is an upper semicontinuous function of z and a measurable function of t. Hence /6/, the solution $z(t)$, $\nu(t)$ of inclusion (2.2) exists in the interval $[\tau, p]$.

Strategy (2.1) will guarantee that terminal set (1.2) is hit at instant p from the initial state z, ν, τ, if, for any control $|v(t)| \leqslant 1$ and any solution of inclusion (2.2), the condition $(z(p), \nu(p)) \in X$ is satisfied. Recalling that the function φ is monotonic, we can write the termination condition as

$$|z(p)| \leqslant \varphi(\nu_p), \quad \nu_p = \nu - \int_\tau^p w(t)\, dt \leqslant \nu(p) \tag{2.3}$$

Let the absolutely continuous function $z(t)$, $\tau \leqslant t \leqslant p$, be a solution of inclusion (2.1). Since the function $|z|$ satisfies a Lipschitz condition, the norm $|z(t)|$ is also an absolutely continuous function. Hence the derivative $|z(t)|^{\cdot}$ exists almost everywhere and /6/

$$|z(t)|^{\cdot} = \lim h^{-1} (|z(t) + hz^{\cdot}(t)| - |z(t)|), \quad h \to 0, \quad h > 0 \tag{2.4}$$

The set of points $\tau \leqslant t \leqslant p$, where $|z(t)| = 0$ and $|z(t)|^{\cdot} = 0$, is not more than denumerable. Hence all the points $t \in [\tau, p]$, at which the derivatives $z^{\cdot}(t)$ and $|z(t)|^{\cdot}$ exist, can be divided into two classes

$$I_1 = \{t\colon |z(t)| > 0\}, \ I_2 = \{t\colon |z(t)| = 0, \ |z(t)|^{\cdot} = 0\} \tag{2.5}$$

The measure of the union of sets (2.5) is equal to the measure of the interval $[\tau, p]$. It follows from (2.4) that, given any solution of inclusion (2.1), we have the inequality

$$|z(t)|^{\cdot} \leqslant -\alpha(t)\, w(t) + \beta(t), \quad t \in I_1 \tag{2.6}$$

We take the measurable function $0 \leqslant w(t) \leqslant \gamma$ for $\tau \leqslant t \leqslant p$ and the number a. We put

$$F(t, z) = |z| - a(t), \quad a(t) = \int_t^p (\alpha(r)\, w(r) - \beta(r))\, dr + a \tag{2.7}$$

With the chosen control $v(t)$ we consider the solution $z(t)$ of the first inclusion of (2.2).

Lemma 1. Let $a(t) \geqslant 0$ for $\tau \leqslant t \leqslant p$ and $F(\tau, z) \leqslant 0$. Then, for all $\tau \leqslant t \leqslant p$,

$$F(t, z(t)) \leqslant 0 \tag{2.8}$$

Proof. The function F of (2.7) satisfies a Lipschitz condition with respect to the set of its variables. Hence the function $F(t, z(t)) = f(t)$ is absolutely continuous. From (2.5), (2.6), and (2.7), we have

$$f'(t) \leqslant 0, \ |z(t)| > 0; \ f'(t) = \alpha(t) w(t) - \beta(t), \ \ |z(t)| = 0 \tag{2.9}$$

It follows from the condition $a(t) \geqslant 0$ that, if $|z(t)| = 0$, then (2.8) is satisfied.

Let $|z(t)| > 0$. Then, by the condition $F(\tau, z) \leqslant 0$, there exists a number $t_0 > t$ such that $f(t_0) \leqslant 0$ and $|z(r)| > 0$ for $t_0 < r \leqslant t$. From this and (2.9) we obtain (2.8).

It follows from (1.7) that, if the function α decreases for $\tau \leqslant t \leqslant p$, then (1.6) takes the form

$$\psi(\tau, \nu) = \max_s \Big(\varphi(\nu - \gamma(s - \tau)) + \gamma \int_\tau^s \alpha(r)\, dr\Big) - \int_\tau^p \beta(r)\, dr \tag{2.10}$$
$$\tau \leqslant s \leqslant \min(p; \ \tau + (\nu - \delta)/\gamma)$$

Let the maximum be reached in (2.10) with s_1. We put

$$M(\tau, \nu) = \varphi(\nu - \gamma(s_1 - \tau)) - \int_{s_1}^p \beta(r)\, dr \tag{2.11}$$

Theorem 1. Let $|z| \leqslant \psi(\tau, \nu)$ and let one of the following hold for $\tau \leqslant t \leqslant p$:

$$\beta(t) = \gamma\alpha(t) \tag{2.12}$$

$$\varphi(\delta) \geqslant \int_t^p \beta(r)\, dr \tag{2.13}$$

$$\gamma\alpha(t) \geqslant \beta(t); \quad t_1 < t_2 \Rightarrow \alpha(t_1) \leqslant \alpha(t_2) \tag{2.14}$$

$$\gamma\alpha(t) \geqslant \beta(t); \quad t_1 < t_2 \Rightarrow \alpha(t_1) > \alpha(t_2); \quad M(\tau, \nu) \geqslant 0 \tag{2.15}$$

Then, there is a function $0 \leqslant w(t) \leqslant \gamma$, measurable for $\tau \leqslant t \leqslant p$, such that, given any measurable control $|v(t)| \leqslant 1$, any solution of inclusion (2.2) satisfies condition (2.3).

Proof. It follows from (1.6) that there is a number $0 \leqslant \mu \leqslant \nu - \delta$, for which

$$|z| \leqslant f(p, \tau, \mu) - \int_\tau^p \beta(r)\, dr + a, \quad a = \varphi(\nu - \mu) \tag{2.16}$$

As $w(t)$ in inclusion (2.2) and in the definition of the function (2.7) we take the solution of problem (1.4) with $\nu = \mu$. Then, from (2.16), $F(\tau, z) \leqslant 0$ and $a(\tau) \geqslant 0$. If we can show that $a(t) \geqslant 0$ for $\tau \leqslant t \leqslant p$, then inequality (2.8) will hold for $t = p$. This inequality, by (2.7) and the definition (2.16) of a, takes the form (2.3).

Let (2.12) hold. Then the function (2.7) $a(t) \geqslant a(\tau) \geqslant 0$ for $\tau \leqslant t$.

If (2.13) holds, then, in view of form (2.16) of the number a, and the fact that φ is not decreasing, the function (2.7) $a(t) \geqslant 0$ for $\tau \leqslant t$.

Let condition (2.14) hold. Then, from (1.4), $w(t) = 0$ for $\tau \leqslant t \leqslant t_0$ and $w(t) = \gamma$ for $t_0 \leqslant t \leqslant p$, where $t_0 = \max(\tau; \ p - \mu/\gamma)$. Consequently, $a'(t) = \beta(t) \geqslant 0$ for $\tau \leqslant t \leqslant t_0$ and $a'(t) = -\gamma\alpha(t) + \beta(t) \leqslant 0$ for $t_0 \leqslant t \leqslant p$. From this and the condition $a(\tau) \geqslant 0$, $a(p) \geqslant 0$, we have $a(t) \geqslant 0$ for $\tau \leqslant t \leqslant p$.

Let (2.15) hold and let the maximum in (2.10) be reached at the point s_1. Then, $w(t) = \gamma$ for $\tau \leqslant t \leqslant s_1$ and $w(t) = 0$ for $s_1 \leqslant t \leqslant p$. Consequently, $a'(t) = -\gamma\alpha(t) + \beta(t) \leqslant 0$ for $t \leqslant s_1$ and $a'(t) = \beta(t) \geqslant 0$ for $s_1 < t$. Hence the minimum value is $a(s_1) = M(\tau, \nu) \geqslant 0$.

Lemma 2. Let the first condition (2.15) hold and let $\nu \geqslant \delta + \gamma(p - \tau)$. Then, $M(\tau, \nu) \geqslant 0$.

Proof. The number (2.10) is not less than the value of the right-hand side of (2.7) with $s = p$. Hence, using the first condition (2.15) and the fact that φ is non-negative, we obtain

$$\varphi(\nu - \gamma(s_1 - \tau)) \geqslant \gamma \int_{s_1}^p \alpha(r)\, dr \geqslant \int_{s_1}^p \beta(r)\, dr$$

3. Consider the case when

$$\nu < \delta + \gamma(p - \tau) \tag{3.1}$$

We assume that, for $\tau \leqslant t \leqslant p$, the first two conditions (2.15) hold and

$$\varphi(\mu) \leqslant \alpha(p)(\mu - \delta), \quad \delta \leqslant \mu < \delta + \gamma(p - \tau) \tag{3.2}$$

We introduce the function

$$B(t,s)=\gamma(s-t)+\int\limits_s^p \frac{\beta(r)}{\alpha(r)}\,dr, \quad t\leqslant s \tag{3.3}$$

Lemma 3. Let the initial position z, ν be such that

$$|z|>\alpha(\tau)(\nu-B(\tau,\tau)-\delta) \tag{3.4}$$

Then the second player, by making a finite number of corrections to his control, can prevent the set (1.2) being hit at the instant p.

Proof. By inequality (3.4), numbers $\tau=t_0<t_1<\ldots<t_{k+1}=p$ exist, such that

$$|z|>\alpha(t_0)\Big(\nu-\sum_{i=0}^{k}A_i-\delta\Big), \qquad A_i=\frac{1}{\alpha(t_i)}\int\limits_{t_i}^{t_{i+1}}\beta(r)\,dr \tag{3.5}$$

When $t_0\leqslant t\leqslant t_1$, the second player takes control (1.9). Then, from (1.10) and the condition that the function α be monotonic (2.15), given any first player's control $|u(t)|\leqslant\gamma$,

$$|z(t_1)|>\alpha(t_1)\Big(\nu(t_1)-\sum_{i=1}^{k}A_i-\delta\Big)$$

Using the vector $z(t_1)$, the second player constructs control (1.9) with $t_1<t\leqslant t_2$, etc.

At the instant of termination $t_{k+1}=p$, the inequality $|z(p)|>\alpha(p)\cdot(\nu(p)-\delta)$ will hold. Hence, using inequality (3.2), we see that the position $z(p)$, $\nu(p)$ does not belong to set (1.2).

In view of this lemma, we need to consider the case

$$\nu\geqslant\delta+\int\limits_{\tau}^{p}\frac{\beta(r)}{\alpha(r)}\,dr=\delta+B(\tau,\tau) \tag{3.6}$$

In accordance with the first condition (2.15), the derivative of function (3.3) with respect to the variable s is non-negative, i.e., the function is not decreasing with respect to s. Consequently, for any ν satisfying (3.1) and (3.6), we can define the number

$$s(\nu)=\max\{s\colon \tau\leqslant s\leqslant p,\ \nu=\delta+B(\tau,\ s)\} \tag{3.7}$$

We put

$$\psi_1(t,\nu)=\int\limits_{\tau}^{s(\nu)}(\gamma\alpha(r)-\beta(r))\,dr \tag{3.8}$$

Theorem 2. Let the initial position z, ν satisfy the condition $|z|\leqslant\psi_1(\tau,\nu)$. Then, there is a function $0\leqslant w(t)\leqslant\gamma$, measurable for $\tau\leqslant t\leqslant p$, such that, with any measurable control $|v(t)|\leqslant 1$, any solution of inclusion (2.2) satisfies condition (2.3).

Proof. We put

$$w(t)=\gamma,\quad \tau\leqslant t\leqslant s(\nu);\quad w(t)=\beta(t)/\alpha(t),\quad s(\nu)<t\leqslant p \tag{3.9}$$

Then, by (3.8), $|z|\leqslant a(\tau)$, where $a(t)$ is given by (2.7) with $a=0$. It follows from the first condition (2.15) that, for the function (3.9), $a(t)\geqslant 0$ for $\tau\leqslant t\leqslant p$. By Lemma 1 we obtain $|z(p)|=0$. For function (3.9), the number (2.3) $\nu_p=\delta$. From this and the inequality $\varphi(\delta)\geqslant 0$ we obtain condition (2.3).

Theorem 3. Let the initial position z, ν satisfy the condition $|z|>\psi_1(\tau,\ \nu)$. Then, the second player, by making a finite number of corrections to his control, can prevent set (1.2) being hit at the instant p.

Proof. If we have the equality in (3.6), then the initial position satisfies condition (3.4).

Let us have the strict inequality in (3.6). Then, $\tau<s<p$, where we put $s=s(\nu)$. The second player takes control (1.9) for $\tau\leqslant t\leqslant s$. By (1.10) and the condition that the function α be monotonic, we find that, with any first player's admissible control, $|z(s)|>\alpha(s)(\gamma(s-\tau)-\nu+\nu(s))$. From this and expression (3.7) for s, we have $|z(s)|>\alpha(s)\cdot(\nu(s)-B(s,s)-\delta)$. By Lemma 3, the second player, on continuing the game from point $z(s)$, $\nu(s)$, can prevent set (1.2) being hit at instant p.

We will evaluate function (2.10) on the assumption that $\varphi(\delta)=0$ and

$$\varphi(\nu_1) - \varphi(\nu_2) \leqslant \alpha(p)(\nu_1 - \nu_2), \quad \delta \leqslant \nu_2 < \nu_1 \tag{3.10}$$

Then, inequality (3.2) is satisfied. Denote by $\varepsilon(s)$ the function whose maximum value is calculated in (2.10). Then, it follows from (3.10) and (2.15) that, with $s_1 < s_2$

$$\varepsilon(s_1) - \varepsilon(s_2) \leqslant \alpha(p)(s_2 - s_1)\gamma - \gamma\int_{s_1}^{s_2}\alpha(r)\,dr \leqslant 0$$

Hence, with $\nu < \delta + \gamma(p - \tau)$

$$\psi(\tau, \nu) = \gamma\int_{\tau}^{\theta}\alpha(r)\,dr - \int_{\tau}^{\theta}\beta(r)\,dr, \quad \theta = \tau + \frac{\nu - \delta}{\gamma} \tag{3.11}$$

It follows from (3.7) that

$$\theta = s + \int_{s}^{p}\frac{\beta(r)}{\alpha(r)}\,dr, \quad s = s^{\cdot}(\nu)$$

We can see from this that function (3.8) and (3.11) satisfy the inequality $\psi_1(\tau, \nu) < \psi(\tau, \nu)$. In the present case, therefore, the family of sets $W(\tau) = T_\tau^p(X)$ is not a stable bridge /3/, leading to target (1.2).

4. *Example.* We consider the problem on the encounter $(|x(q) - y(q)| \leqslant a)$ at a given instant q of a point moving with bounded velocity $|y^{\cdot}| \leqslant \beta$, with a point of variable state whose motion is described by Meshcherskii's equation /5/ $x^{\cdot\cdot} = b + wm^{\cdot}(t)/m(t)$. Here, b is a constant vector characterizing the external force; $m(t) = m_0 + m_1(t)$ is the mass, where m_0 is the fixed part of the mass, and $m_1(t)$ is the reactive mass; w is the relative velocity separating the particles, the magnitude $|w|$ of which is assumed constant.

We put

$$z = y - x - (q - t)x^{\cdot} + b(q - t)^2/2, \quad v = y^{\cdot}/\beta$$
$$u = -wm^{\cdot}(t)/m(t), \quad \nu(t) = |w|\ln(m(t)/m_0) \tag{4.1}$$

Then, the encounter condition and the equations of motion take the form

$$|z(q)| \leqslant a; \quad z^{\cdot} = -(q - t)u + \beta v, \quad |v| \leqslant 1, \quad \nu^{\cdot} = -|u| \tag{4.2}$$

The inequality $\nu(t) \geqslant 0$ means that the reactive mass is non-negative. We assume that the thrust is bounded: $|u| \leqslant \gamma$.

The first player, who chooses control u, tries to realize an encounter. The second player, who chooses control v, has the opposite aim.

In this game, the function $\alpha(t) = q - t$, $\beta(t) = \beta$, and in the terminal set (1.2) $\varphi(\nu) = a$, $\delta = 0$.

We write the function (2.10)

$$\psi(\tau, \nu) = \gamma(c(\tau) - (q - r)^2)/2$$
$$c(\tau) = (q - \tau)^2 - 2(\beta(q - \tau) + a)/\gamma, \quad r = \min(q; \tau + \nu/\gamma) \tag{4.3}$$

Put

$$l = \gamma\max(a/\beta; \beta/\gamma), \quad p = q - l/\gamma \tag{4.4}$$

Then, when $p \leqslant t \leqslant q$, either condition (2.12) or (2.13) holds. Thus, for initial instants $\tau \in [p, q]$, Theorem 1 is applicable. The function $w(t)$ in (2.2), guaranteeing an encounter, is: $w(t) = \gamma$ for $\tau \leqslant t \leqslant r$ and $w(t) = 0$, $t > r$. From this and (4.1) and (2.1) we obtain the relative velocity and the law of mass variation

$$w = -|w|(z/|z|), \quad m(t) = m(\tau)\exp(-\gamma(t - \tau)/|w|) \tag{4.5}$$

Consider the case when $a < \beta^2/(2\gamma)$. Then, $\psi(\tau, \nu) < 0$ for all $\nu > 0$ and $p \leqslant \tau < \tau(a) = q - (\beta - (\beta^2 - 2a\gamma)^{1/2})/\gamma$. The set (1.5) is empty for these τ Hence /7/, for an initial instant $\tau < \tau(a)$ and any initial position, there is a second player's strategy such that the first player cannot realize an encounter.

Let $a \geqslant \beta^2/(2\gamma)$. Then, $c(\tau) \geqslant 0$ for all τ and set (1.5) is not empty for $p \leqslant \tau \leqslant q$.

To write the condition for the termination of the game which starts at an initial instant $\tau < p$, we have to consider the problem of hitting at instant p the terminal set (1.2) with function $\varphi(\nu) = \psi(p, \nu)$. From (4.3) and (4.4) we have

$$\varphi(\nu) = (l - \delta)^2/(2\gamma) - (\max(0; l - \nu))^2/(2\gamma)$$
$$\nu \geqslant \delta = l - (l^2 - 2\beta l + 2a\gamma)^{1/2}$$

The derivative of the function φ is bounded by $l/\gamma = \alpha(p)$. Consequently, conditions (3.10) are satisfied.

By Lemma 2 and Theorem 1, with $\nu \geqslant \delta + (p - \tau)\gamma = \delta + (q - \tau)\gamma - l$ the control which guarantees an encounter is given by (4.5).

Let inequalities (3.1) and (3.6) hold, i.e.,

$$\delta+\beta \ln\left((q-\tau)\frac{\gamma}{l}\right)\leqslant \nu<\delta+(q-\tau)\gamma-l$$

We write the equation for the number s of (3.7):

$$\nu=\delta+(s-\tau)\gamma+\beta\ln((q-s)\gamma/l)$$

Then, we find from (3.9) that the law of mass variation has the form (4.5) with $\tau\leqslant t\leqslant s$, and with $s\leqslant t\leqslant p$,

$$m(t)=m(s)((q-t)/(q-s))^k,\quad k=\beta/|w|$$

Knowing the conditions for an encounter with any $a\geqslant 0$, we can find /4/ the value of the game, when the pay-off is the distance $|z(q)|$. In our example, the set $T_t^q(X)$ is not a stable bridge. This implies that termination of the game after the first instant of absorption /8/ is not possible for all initial positions, while the value of the game is not the same as the programmed max-min.

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CONSTRUCTION OF MIXED STRATEGIES ON THE BASIS OF STOCHASTIC PROGRAMS*

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The optimal control problem in the class of mixed strategies is considered, under the condition that the guaranteed result is minimized. An efficient method of constructing the optimal strategy by means of stochastic program synthesis is given. The results extend the theory given in /1-7/.

1. Formulation of the problem. We consider the object described by the differential equation

$$x^{\cdot}=A(t)x+f(t,u,v),\quad t_0\leqslant t\leqslant\vartheta,\quad u\in R,\quad v\in W \tag{1.1}$$

where x is the n-dimensional phase vector, u the r-dimensional control vector, v the s-dimensional noise vector, R and W are compacta, the matrix function $A(t)$ and vector function

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